

Young's Double Slit Experiment in Quantum Field Theory

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Abstract

Young's double slit experiment is formulated in the framework of canonical quantum field theory in view of the modern quantum optics. We adopt quantum scalar fields instead of quantum electromagnetic fields ignoring the vector freedom in gauge theory. The double slit state is introduced in Fock space corresponding to experimental setup. As observables, expectation values of energy density and positive frequency part of current with respect to the double slit state are calculated which give the interference term. Classical wave states are realized by coherent double slit states in Fock space which connect quantum particle states with classical wave states systematically. In case of incoherent sources, the interference term vanishes by averaging random phase angles as expected.

1 Introduction

In the early nineteen century, Thomas Young performed his famous double slit diffraction experiment using a light source, which shows the interference pattern on the screen [1]. The interference effect is explained in the framework of classical optics by the method of Huygens-Fresnel principle in classical optics theory [2]. As a modern version of this experiment, the quantum double slit experiments are performed using photons, electrons, neutrons and others. It is necessary to use quantum mechanics in order to explain the interference pattern for subatomic particles [3].

In standard quantum mechanics, the double slit wave function is obtained as a special solution of the Schrödinger wave equation as the superposition of waves emitted from slits A, B at the points $(d/2, 0, 0)$, $(-d/2, 0, 0)$

$$F_k^{DS}(\mathbf{r}, t) = F_k^A(\mathbf{r}, t) + F_k^B(\mathbf{r}, t), \quad (1.1)$$

where each solution is expressed by spherical waves as a good approximation in the region between slits and screen:

$$F_k^A(\mathbf{r}, t) = A_0 \exp(-i\omega_k t + ikr_A)/r_A, \quad F_k^B(\mathbf{r}, t) = B_0 \exp(-i\omega_k t + ikr_B)/r_B, \quad (1.2)$$

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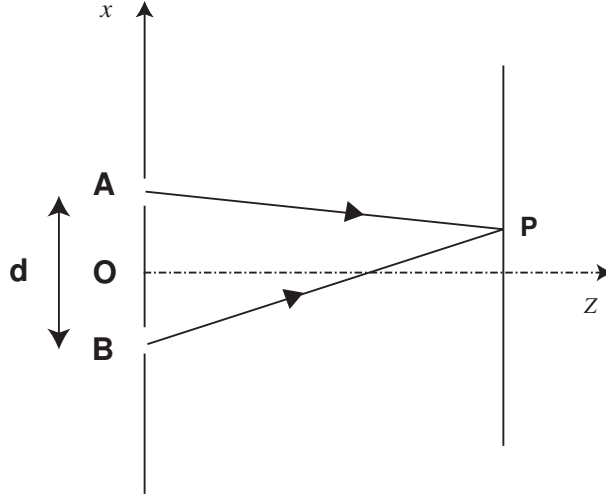


Figure 1: Schematic design of Young's double slit experiment between slits and screen. Coordinates are assigned as the origin $O = (0, 0, 0)$, slits $A=(d/2,0,0)$, $B=(-d/2,0,0)$ and observation position $P = (x, y, z)$.

where $\mathbf{r} = (x, y, z)$ denotes the observation point on the screen and r_A, r_B are distances between each slit and the observation point:

$$r_A = \sqrt{(x - d/2)^2 + y^2 + z^2}, \quad r_B = \sqrt{(x + d/2)^2 + y^2 + z^2}. \quad (1.3)$$

The wave number and frequency are denoted by $k = (k_x, k_y, k_z)$ with $k = |k|$ and ω_k . Amplitudes include fixed absolute values and phases for double-sources: $A_0 = |A_0| \exp i\theta_A$, $B_0 = |B_0| \exp i\theta_B$. Intensity of wave function is given as

$$|F_k^{DS}(\mathbf{r}, t)|^2 = |F_k^A(\mathbf{r}, t)|^2 + |F_k^B(\mathbf{r}, t)|^2 + 2 \operatorname{Re}(F_k^A(\mathbf{r}, t)^* F_k^B(\mathbf{r}, t)), \quad (1.4)$$

where the interference effect appears in the last term as

$$\begin{aligned} 2 \operatorname{Re}(F_k^A(\mathbf{r}, t)^* F_k^B(\mathbf{r}, t)) &= 2 \frac{|A_0||B_0|}{r_A r_B} \cos(k(r_A - r_B) + \theta_A - \theta_B) \\ &\simeq 2 \frac{|A_0||B_0|}{r^2} \cos(-kxd/r + \theta_A - \theta_B), \end{aligned} \quad (1.5)$$

which holds for $d \ll r = \sqrt{x^2 + y^2 + z^2}$. The intensity is interpreted as the probability density in the non-relativistic Schrödinger theory because it is the density of the conserved current. We introduce each phase for each source in order to treat incoherent cases as well as coherent cases. We are able to obtain the same interference term for the classical wave mechanics including classical electromagnetism as the non-relativistic quantum mechanics.

Photons and particles in high speed, however, should be treated in the framework of quantum theory with relativistic covariance. The relativistic quantum mechanics is not complete

theory because of some difficulties: the existence of negative-energy solutions, the lack of probability interpretation in Klein-Gordon theory, the non-existence of charge conjugation states in Dirac theory [4]. Therefore relativistic particles should be treated as the relativistic quantum field theory taking account of the particle creation and annihilation [5].

Several works have been done in this direction, where radiation waves from sources A and B are described by two spherical waves in quantum field theoretical treatment but on the other hand those are described by plane-waves in classical treatment [6, 7]. In this paper, we study the method that the radiation waves from two sources are treated as two spherical waves in quantum field theory. Our method shows that the spherical wave states, A and B, are not independent $\langle A|B \rangle \neq 0$ different from the plane-wave treatment as $\langle A|B \rangle = 0$, and can naturally connect the description of Young's double slit experiment in quantum field theory with that in quantum mechanics and in classical wave mechanics.¹

We formulate the Young's double slit experiment in standard canonical formalism of quantum field theory by introducing the double slit states in Fock space. Our basic idea is that quantum fields are treated in general canonical form and state vectors are in a special form corresponding to the experimental setup or boundary condition by introducing the double slit states in Fock space. We formulate it in the quantum scalar field theory instead of the quantum electrodynamics avoiding the complexity arising from consistent compatibility of gauge invariance with relativistic covariance. We are able to understand the relation in the interference effects among classical wave mechanics, quantum mechanics and quantum field theory by studying the quantum field theory through the coherent state method [9].

The organization of this paper is the following. In section 2, the general description is shown for the Young's double slit experiment in the relativistic quantum field theory for massive and/or massless scalar field. In subsection 2.1 the canonical quantization for quantum scalar field is reviewed for the sake of following study. In subsection 2.2 general formalism for double-source states is introduced in Fock space. In subsection 2.3 application of the general formalism to the Young's double slit experiment is shown. In section 3 classical waves are realized by introducing the coherent double slit states in Fock space. In section 4 case of incoherent sources are studied. In the final section, the result is summarized and some discussions are given.

2 Double slit experiment in scalar field theory

Field theoretical approach to the double slit problem is formulated for field operators as general solutions of operator field equations and for state vectors in Fock space connecting special solutions of c-number field equations. The Heisenberg picture is taken in the following.

2.1 Canonical scalar field theory

In this subsection, we review and summarize quantum field theory for the following convenience. Here we take quantum free scalar theory because we mainly consider their propagation

¹ The plane-wave treatment can reproduce the same interference effect as the classical wave mechanics but cannot express the global structure of space as the distance dependence on sources and the screen.

in free space between slits and screen. We also ignore the helicity freedom of electromagnetic fields because we focus on the interference effects not special to each helicity state.

We start from the action and the Lagrangian for free scalar quantum field theory in the natural unit: $c = \hbar = 1$ as

$$I = \int dt L(t) , \quad (2.1)$$

$$L(t) = \int d^3x ((\dot{\Phi}(\mathbf{r}, t))^2 - (\nabla\Phi(\mathbf{r}, t))^2 - \mu^2\Phi(\mathbf{r}, t)^2)/2 , \quad (2.2)$$

where $\dot{\Phi}$ and μ denote the time derivative and the mass term for the scalar particle. The canonical momentum with respect to the scalar field is defined and obtained:

$$\Pi(\mathbf{r}, t) = \delta L(t)/\delta\dot{\Phi}(\mathbf{r}, t) = \dot{\Phi}(\mathbf{r}, t) , \quad (2.3)$$

and the canonical equal-time commutation relations are imposed:

$$[\Phi(\mathbf{r}, t), \Pi(\mathbf{r}', t)] = i\delta^{(3)}(\mathbf{r} - \mathbf{r}') , [\Phi(\mathbf{r}, t), \Phi(\mathbf{r}', t)] = [\Pi(\mathbf{r}, t), \Pi(\mathbf{r}', t)] = 0 . \quad (2.4)$$

The Hamiltonian and Hamiltonian density are defined as

$$H = \int d^3x \Pi(\mathbf{r}, t)\dot{\Phi}(\mathbf{r}, t) - L = \int d^3x \mathcal{H}(\mathbf{r}, t) , \quad (2.5)$$

$$\mathcal{H}(\mathbf{r}, t) = : (\Pi(\mathbf{r}, t)^2 + (\nabla\Phi(\mathbf{r}, t))^2 + \mu^2\Phi(\mathbf{r}, t)^2) : /2 , \quad (2.6)$$

where the mark $:$ denotes the normal order product. Using the Heisenberg equation of motion for $\Phi(\mathbf{r}, t)$ and $\Pi(\mathbf{r}, t)$, the field equation for quantum fields is obtained:

$$\ddot{\Phi}(\mathbf{r}, t) - \nabla^2\Phi(\mathbf{r}, t) - \mu^2\Phi(\mathbf{r}, t) = 0 . \quad (2.7)$$

The general solution of the operator field equation is given by the expansion of eigen-value solutions with operator coefficients $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$:

$$\Phi(\mathbf{r}, t) = \sum_{\mathbf{k}} (u_{\mathbf{k}}(\mathbf{r}, t)a_{\mathbf{k}} + u_{\mathbf{k}}^*(\mathbf{r}, t)a_{\mathbf{k}}^\dagger) , \quad (2.8)$$

where the plane wave solutions are taken as eigen-value solutions with the periodic boundary condition:

$$u_{\mathbf{k}}(\mathbf{r}, t) = n_{\mathbf{k}} \exp(-i\omega_{\mathbf{k}}t + i\mathbf{k} \cdot \mathbf{r}) , \quad (2.9)$$

where the wave number, frequency and the normalization factor are denoted as $\mathbf{k} = (k_x, k_y, k_z)$ with $k = |\mathbf{k}|$, $\omega = \sqrt{k^2 + \mu^2}$ and $n_{\mathbf{k}} = 1/\sqrt{2\omega_{\mathbf{k}}V}$ (V : the volume of space) respectively. They satisfy the ortho-normal relations:

$$(u_{\mathbf{k}}, u_{\ell}) = -(u_{\mathbf{k}}^*, u_{\ell}^*) = \delta_{\mathbf{k}, \ell} , (u_{\mathbf{k}}^*, u_{\ell}) = (u_{\mathbf{k}}, u_{\ell}^*) = 0 , \quad (2.10)$$

where the relativistic inner product is defined:

$$(A, B) = i \int d^3x (A^* \dot{B} - \dot{A}^* B) , \quad (2.11)$$

with the completeness relation: ²

$$i \sum_{\mathbf{k}} (u_{\mathbf{k}}^*(\mathbf{r}, t) \dot{u}_{\mathbf{k}}(\mathbf{r}', t) - \dot{u}_{\mathbf{k}}^*(\mathbf{r}, t) u_{\mathbf{k}}(\mathbf{r}', t)) = \delta^{(3)}(\mathbf{r} - \mathbf{r}') . \quad (2.12)$$

The commutation relations among $a_{\mathbf{k}}, a_{\mathbf{k}}^\dagger$ are obtained from the canonical commutation relations in eq.(2.4) and the ortho-normal relations in eq.(2.10) as

$$[a_{\mathbf{k}}, a_{\ell}^\dagger] = \delta_{\mathbf{k}, \ell} , \quad [a_{\mathbf{k}}, a_{\ell}] = [a_{\mathbf{k}}^\dagger, a_{\ell}^\dagger] = 0 . \quad (2.13)$$

The Hamiltonian is expressed by the creation and annihilation operators omitting the zero point energy:

$$H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} . \quad (2.14)$$

2.2 General formalism for state vector

In this subsection, we develop the method to form the state vector in the Fock space for the general interference experiment in the Heisenberg picture. Our method can connect solutions of the quantum field theory with those of quantum mechanics and of classical wave mechanics.

We start to consider a state $|X\rangle$ which is the superposition of one particle state in the Fock space:

$$|X\rangle = \sum_{\ell} f_{\ell} a_{\ell}^\dagger |0\rangle , \quad (2.15)$$

with the superposition coefficients f_{ℓ} . In order to determine the superposition coefficients f_{ℓ} , the c-number function $F(\mathbf{r}, t)$ of one particle expectation value is introduced:

$$F(\mathbf{r}, t) = \langle 0 | \Phi(\mathbf{r}, t) | X \rangle = \sum_{\ell} f_{\ell} u_{\ell} , \quad (2.16)$$

where u_{ℓ} is the eigen-value solution of eq.(2.9). ³ As the quantum field $\Phi(\mathbf{r}, t)$ is the general solution of field equation eq.(2.7), the function $F(\mathbf{r}, t)$ should satisfy the c-number field equation in the same form:

$$\ddot{F}(\mathbf{r}, t) - \nabla^2 F(\mathbf{r}, t) - \mu^2 F(\mathbf{r}, t) = 0 . \quad (2.17)$$

It should be noted that some source terms and/or boundary terms can be taken into account in this c-number field equation corresponding to the experimental setup. If the special solution $F(\mathbf{r}, t)$ is obtained, the superposition coefficients f_{ℓ} are obtained as the Fourier transformation coefficients:

$$f_{\ell} = (u_{\ell}(\mathbf{r}, t), F(\mathbf{r}, t))|_{t=0} , \quad (2.18)$$

² In the completeness relation, prescription $i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}') \rightarrow i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}') - \epsilon|\mathbf{k}|$ is to be understood, with the small and positive regularization parameter ϵ .

³ If the normalization condition is imposed on the state $\langle X | X \rangle = 1$, the superposition coefficients satisfy the condition $\sum_{\ell} f_{\ell}^* f_{\ell} = 1$. Under this condition, the one particle expectation value is also normalized ($F(\mathbf{r}, t), F(\mathbf{r}, t) = 1$).

where the round bracket (A, B) denotes the relativistic inner product defined in eq.(2.11) and thus the corresponding state vector in the Fock space is determined as in eq.(2.15). In the end of the calculation in eq.(2.18), the time is set to zero because the Heisenberg picture is taken.

2.3 Double slit state

As an important application of this general method to form the special state in the Fock space, we study the state vectors for the Young's double slit experiment.

In order to obtain the double slit state $|DS; k\rangle$, firstly we solve the one particle c-number field equation of eq.(2.17) and the spherical wave solution $F_k^{DS}(\mathbf{r}, t) = F_k^A(\mathbf{r}, t) + F_k^B(\mathbf{r}, t)$ is obtained as the special solution as in the non-relativistic quantum mechanics of eqs.(1.1) and (1.2). The superposition coefficients $f_{\ell,k}^{DS}$ are obtained as the Fourier expansion coefficients of the c-number double-source function $F_k^{DS}(\mathbf{r}, t)$:⁴

$$\begin{aligned} f_{\ell,k}^{DS} &= f_{\ell,k}^A + f_{\ell,k}^B, \\ f_{\ell,k}^A &= (u_\ell, F_k^A / \sqrt{2\omega_k})|_{t=0} = \frac{4\pi n_\ell(\omega_\ell + \omega_k)}{\sqrt{2\omega_k}(\ell^2 - k^2)} A_0 \exp(-i\ell_x d/2), \\ f_{\ell,k}^B &= (u_\ell, F_k^B / \sqrt{2\omega_k})|_{t=0} = \frac{4\pi n_\ell(\omega_\ell + \omega_k)}{\sqrt{2\omega_k}(\ell^2 - k^2)} B_0 \exp(i\ell_x d/2), \end{aligned} \quad (2.19)$$

where $\ell = (\ell_x, \ell_y, \ell_z)$ is the linear momentum and n_ℓ is the normalization factor of the plane waves given below the eq.(2.9). At the end of calculation, the time variable for the double slit state is set to be zero because the Heisenberg picture is taken. Additional factor $\sqrt{2\omega_k}$ is included for the normalization of relativistic inner product. The double-source state $|DS; k\rangle$ for a fixed wave number $k = |\mathbf{k}|$ is formed in Fock space with the superposition coefficients $f_{\ell,k}$:

$$|DS; k\rangle = \sum_{\ell} a_{\ell}^{\dagger} |0\rangle f_{\ell,k}^{DS}. \quad (2.20)$$

We can confirm that one particle expectation value on the screen position \mathbf{r}

$$\langle 0 | \Phi(\mathbf{r}, t) | DS; k \rangle = (F_k^A(\mathbf{r}, t) + F_k^B(\mathbf{r}, t)) / \sqrt{2\omega_k} = F_k^{DS}(\mathbf{r}, t) / \sqrt{2\omega_k}, \quad (2.21)$$

reproduces the same spherical wave function in quantum mechanics or classical wave mechanics for the double-slit experiment.⁵

Note that the one-particle expectation value $\langle 0 | \Phi(\mathbf{r}, t) | DS; k \rangle$ in eq.(2.21) does not have the meaning of provability amplitude of wave function because of the non-existence of the consistent relativistic quantum mechanics but plays the role of provability density for conserved quantities. For the case that the positive $\Phi^{(+)}(\mathbf{r}, t)$ and negative $\Phi^{(-)}(\mathbf{r}, t)$ frequency parts of the scalar fields can be separated definitely, which is possible for the free field theory, the conserved current for $\Phi^{(+)}(\mathbf{r}, t)$ can be constructed [8]:

$$j_{\mu}^{(+)}(\mathbf{r}, t) = -i(\Phi^{(+)}(\mathbf{r}, t)^{\dagger} \partial_{\mu} \Phi^{(+)}(\mathbf{r}, t) - \partial_{\mu} \Phi^{(+)}(\mathbf{r}, t)^{\dagger} \Phi^{(+)}(\mathbf{r}, t)), \quad (2.22)$$

⁴ In deriving the Fourier expansion coefficient, prescription $k \rightarrow k + i\epsilon$ is to be understood.

⁵ Of course, the frequency ω_k is $\sqrt{k^2 + \mu^2}$ for relativistic theory and $k^2/2\mu$ for non-relativistic theory.

where

$$\Phi^{(+)}(\mathbf{r}, t) = \sum_k u_k(\mathbf{r}, t) a_k . \quad (2.23)$$

Its density is a candidate for good physical observables and the expectation value for the double slit state is calculated as

$$\langle DS; k | j_0^{(+)}(\mathbf{r}, t) | DS; k \rangle = |F_k^{DS}(\mathbf{r}, t)|^2 , \quad (2.24)$$

which is the expected result. Another good candidate for observables is the energy density because the total energy conserves. The expectation value of energy density on the screen position \mathbf{r} with respect to the double-source state is given as ⁶

$$\langle DS; k | \mathcal{H}(\mathbf{r}, t) | DS; k \rangle = \omega_k |F_k^{DS}(\mathbf{r}, t)|^2 + O(1/r^3) , \quad (2.25)$$

which is again the expected form for the double slit experiment.

It should be noted that the density matrix method for the quantum observable is applied to our method as

$$\langle DS; k | O | DS; k \rangle = \text{Tr} \{ \rho_k^{DS} O \} \quad \text{for } O = j_0^{(+)}(\mathbf{r}, t) , \mathcal{H}(\mathbf{r}, t) , \quad (2.26)$$

where the density matrix is $\rho_k^{DS} = |DS; k \rangle \langle DS; k|$. ⁷

We have obtained the same interference pattern for the famous Young's double slit experiment in the quantum field theoretical description of eqs.(2.24) and (2.25) with the quantum mechanical description of eq.(1.5) in our double slit state method given in eq.(2.20).

3 Classical waves as coherent double-source state

In this section, we are going to study the relation between the relativistic quantum field theory and the corresponding classical wave mechanics for the interference experiment.

In order to establish the relation, we introduce the general coherent state $|CX \rangle$ for the c-number wave function $F(\mathbf{r}, t)$ as the eigen-value function of the positive frequency part of the general operator field $\Phi^{(+)}(\mathbf{r}, t)$:

$$\Phi^{(+)}(\mathbf{r}, t) |CX \rangle = F(\mathbf{r}, t) |CX \rangle . \quad (3.1)$$

As the positive frequency part of quantum field $\Phi^{(+)}(\mathbf{r}, t)$ satisfies the field equation in eq.(2.7), the eigen-value function should satisfy the same field equation. Because the positive frequency part contains only annihilation operators a_ℓ , the eigen-state $|CX \rangle$ is given by the exponential function of the superposition of creation operators a_ℓ^\dagger , that is the coherent state, as

$$|CX \rangle = \exp \left(\sum_\ell f_\ell a_\ell^\dagger \right) |0 \rangle . \quad (3.2)$$

⁶ The higher order term $O(1/r^3)$ comes from the calculation of the term $(\nabla \Phi(r, t))^2/2$ in eq.(2.6).

⁷ The double-source state $|DS; k \rangle$ is not a normalized state in general but can be a normalized one: $|DS; k \rangle / \sqrt{\langle DS; k | DS; k \rangle}$.

Using commutation relations in eq.(2.13), the following relations are derived from eqs.(3.2) and (3.1) as

$$a_\ell |CX\rangle = f_\ell |CX\rangle, \quad \sum_\ell f_\ell u_\ell = F(\mathbf{r}, t). \quad (3.3)$$

Then the superposition coefficient f_ℓ is again given by the Fourier transformation coefficient of $F(\mathbf{r}, t)$ given in eq.(2.18). The method establish to make the relation of the general quantum field to special c-number field.

This general method is applied to the case of double slit experiment. The coherent double slit state $|CDS; k\rangle$ is obtained as the coherent state according to the above method as

$$|CDS; k\rangle = \exp\left(\sum_\ell a_\ell^\dagger f_{\ell,k}^{DS}\right) |0\rangle, \quad (3.4)$$

which satisfies the eigen-value equation:

$$\Phi^{(+)}(\mathbf{r}, t) |CDS; k\rangle = F_k^{DS}(\mathbf{r}, t) |CDS; k\rangle, \quad (3.5)$$

where the superposition coefficients are given as the same forms in eq.(2.19). This coherent double slit state is the extended application of the coherent state for the monochromatic state $\exp \alpha a^\dagger$, which satisfies $a \exp \alpha a^\dagger = \alpha \exp \alpha a^\dagger$, to the spherical waves with definite wave length.

One-particle expectation value for the coherent double slit state is given as

$$\langle 0 | \Phi(\mathbf{r}, t) |CDS; k\rangle = (F_k^A(\mathbf{r}, t) + F_k^B(\mathbf{r}, t)) / \sqrt{2\omega_k} = F_k^{DS}(\mathbf{r}, t) / \sqrt{2\omega_k}, \quad (3.6)$$

which is the same as the one for the double slit state in eq.(2.21). The expectation value of the Hamiltonian density for the coherent double slit state is obtained as

$$\langle CDS; k | \mathcal{H}(\mathbf{r}) |CDS; k\rangle / \langle CDS; k |CDS; k\rangle = \omega_k |F_k^{DS}(\mathbf{r}, t)|^2 + O(1/r^3), \quad (3.7)$$

which is also the same as the one for the double slit state in eq.(2.25).

The coherent double slit state is the set of the Poisson distribution for quantum photons and the operation of the annihilation operators to this state makes them to classical Fourier expansion coefficients which correspond to the classical spherical waves. The coherent double slit state method connects classical electromagnetic wave states with quantum photon states, because the coherent double slit state can reduce to one photon state by choosing the values of amplitudes A_0, B_0 small (see eq.(2.19)).

It should be noted that the coherent double slit state method for spherical waves is more general than the usual coherent state method for monochromatic plane waves like laser beams, which is produced by the continuous induced emission process of photons. Some general and interesting investigations on coherent states have been done extensively[9].

4 Case of incoherent sources

We have studied the case of light sources A,B with definite phases θ_A, θ_B for quantum photons in the double slit state and coherent double slit state for classical waves in the previous subsections. Next we study the case of two incoherent sources. In this case the phases for two

sources A and B are incoherent and random with each other and therefore observable values are expected as the average values of them. The energy density and the positive frequency part of the current density in case of incoherent sources are

$$\begin{aligned} \int_0^{2\pi} \frac{d\theta_A}{2\pi} \int_0^{2\pi} \frac{d\theta_B}{2\pi} \langle DS; k | \mathcal{H}(\mathbf{r}) | DS; k \rangle &\simeq \omega_k (|F_k^A(\mathbf{r}, t)|^2 + |F_k^B(\mathbf{r}, t)|^2), \\ \int_0^{2\pi} \frac{d\theta_A}{2\pi} \int_0^{2\pi} \frac{d\theta_B}{2\pi} \langle DS; k | j_0^{(+)}(\mathbf{r}, t) | DS; k \rangle &= |F_k^A(\mathbf{r}, t)|^2 + |F_k^B(\mathbf{r}, t)|^2, \end{aligned} \quad (4.1)$$

where the interference terms disappear. The expectation values for the coherent double slit states $|CDS; k\rangle$, the interference terms are shown to disappear.

Note that our method is quite different from the one that the double slit state in Fock space is formed as the superposition of the independent state of A and B, independent monochromatic wave states $\mathbf{k}_A, \mathbf{k}_B$ or independent spherical wave states. Indeed our spherical states A and B are not independent but their inner product has non-zero value:

$$\langle B; k | A; k \rangle \propto B_0^* A_0 \frac{\sin kd}{kd}, \quad (4.2)$$

where each spherical states are defined

$$|A; k\rangle := \sum_{\ell} a_{\ell}^+ |0\rangle f_{\ell, k}^A, \quad |B; k\rangle := \sum_{\ell} a_{\ell}^+ |0\rangle f_{\ell, k}^B, \quad (4.3)$$

with the Fourier coefficients given in eq.(2.20).⁸ We can confirm that the limiting cases of the inner product of eq.(4.2) coincides the norm itself in short separation limit and tends to zero in infinitely separation limit:

$$\langle B; k | A; k \rangle \rightarrow \begin{cases} \langle A; k | A; k \rangle & \text{for } d \rightarrow 0 \text{ and } B_0 = A_0 \\ 0 & \text{for } d \rightarrow \infty \end{cases}. \quad (4.4)$$

This result confirms that our method is reasonable and consistent for the incoherent sources of interference phenomena.

5 Conclusion and discussions

We have formulated the famous Young's double slit experiment in quantum field theory based on the modern quantum optics by introducing the double slit state in Fock space $|DS; k\rangle$. As good physical observables the expectation values of energy density $\mathcal{H}(\mathbf{r}, t)$ and current density of positive frequency part $j_0^{(+)}(\mathbf{r}, t)$ with respect to the double slit state for quantum photons are calculated and the interference terms are obtained as the classical Young's experiment.

The connection of the quantum field theoretical method with the classical wave mechanical one is established by introducing the coherent double slit state in Fock space $|CDS; k\rangle$, which is the double slit eigen-state of the annihilation operator. The expectation values of physical

⁸ The exact expression of the inner product is $\langle B; k | A; k \rangle = 2\pi B_0^* A_0 \sin kd / \epsilon kd$, where ϵ is the regularization parameter introduced in the calculation of Fourier transformation; see footnote 4.

observables with respect to the coherent double slit state are calculated and the interference terms are obtained as the quantum photon case. Our formulation can connect quantum photon state with classical wave systematically and gives the consistent result.

For the incoherent and independent sources, The incoherent effect is introduced by averaging the phase angle and the interference terms disappear.

Note that in our method of the double slit state in Fock space, single slit states $|A; k\rangle$ and $|B; k\rangle$ are not independent but have the non-zero inner product $\langle B; k|A; k\rangle \neq 0$, which tends to the norm of the state A: $\langle A; k|A; k\rangle$ for zero slit distance ($d \rightarrow 0$) and zero for large slit distance ($d \rightarrow \infty$). This result is thought to be reasonable. Our method is quite different from the one in which the incoherent sources are treated to be independent $\langle B; k|A; k\rangle = 0$, which do not correspond to the real situation of the double slit spherical waves.

The extension of the double slit experiment in our quantum field theory to the quantum electrodynamics is straightforward and the effect of the helicity vectors is interesting [10]. The work will appear in a separate paper. Our method also can be applied to the Hanbury Brown and Twiss intensity interferometry effect [11, 12, 13] and the result will appear in another paper.

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